A Weighted Least Squares Approach to the Design of FIR Filters Synthesized Using the Modified Frequency Response Masking Structure

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Abstract—This paper presents an application of the weighted least squares (WLS) method to the design of sharp linear phase finite-impulse response (FIR) digital filters synthesized using a modified frequency-response masking (FRM) structure. In our approach, the original minimax design problem is converted into a WLS problem. The WLS problem is highly nonlinear with respect to the coefficients of the filter. However, it can be decomposed into four linear least squares (LS) problems, each of which can be solved analytically. The design problem is then solved iteratively by using an alternating variable approach. The effectiveness of the method is demonstrated through solving a low-pass linear phase sharp FIR digital filter example.

Index Terms—Frequency-response masking (FRM), minimax error, weighted least squares (WLS), alternating variable approach.

I. INTRODUCTION

THE frequency-response masking approach is one of the most efficient techniques to synthesize linear-phase finite-impulse response (FIR) digital filters with sharp transition bands [1]–[4]. This approach produces filters with very sparse coefficients and hence reduces the number of multipliers and adders required when compared with those produced by conventional direct design methods. The basic structure of a filter synthesized using the frequency-response masking (FRM) technique is shown in Fig. 1. If $F(z)$ is the z -transform transfer function of the system, then, from Fig. 1

$$F(z) = F_a(z^M)F_{ma}(z) + \left\{ z^{\frac{-M(l-1)}{2}} - F_a(z^M) \right\} F_{mc}(z).$$

Here, $F_a(z^M)$ is the interpolated band-edge shaping filter, which can be obtained by replacing each delay element of a prototype filter, $F_a(z)$, by M delay elements. l is the filter length of $F_a(z)$. $F_{ma}(z)$ and $F_{mc}(z)$ are two masking filters. Let w_p and w_s be the band edges of $F(z)$. Then, the transition width of $F_a(z)$ is $M(w_s - w_p)$, which increases with M . Hence, the complexity of $F_a(z)$ decreases with increasing

Fig. 1. Basic structure used in the FRM technique.

M . However, the sum of the transition widths of $F_{ma}(z)$ and $F_{mc}(z)$ is $(1/M)$, which is decreasing when M increases. If M is very large, both $F_{ma}(z)$ and $F_{mc}(z)$ become sharp filters. Recently, several modified FRM structures, such as those reported in [5]–[16] have been introduced. In particular, the approach proposed in [12] is to replace the band-edge shaping filter $F_a(z^M)$ with an interpolated FIR (IFIR) filter (for a definition, see [17]–[19]) so as to form an IFIR-FRM structure. With this new structure, the requirements on the masking filters are relaxed. However, the drawback of this approach is that the filter length of one of the masking filters becomes excessively long, which is not optimal from a structural point of view. This is because minimum complexity is achieved (see [2], [20]) only when the transition widths (or complexity) of the two masking filters are equal. To address this problem, the IFIR-FRM structure proposed in [12] is further improved in [11]. First, the IFIR-based band-edge shaping filter is generalized to an IFIR alike filter. Secondly, an additional filtering stage is inserted between the band-edge shaping filter and the masking filters so as to decouple the masking filters from the band-edge shaping filter. Hence, the complexity of the overall filter is reduced significantly. However, it is noted that the subfilters are essentially optimized separately in these papers.

This paper introduces a joint optimization method for the design of linear-phase FIR digital filters synthesized using the modified FRM structure introduced in [11]. The method is based on the weighted least squares (WLS) approach. The original least squares (LS) problem is decomposed into four simple linear LS problems. Each of these linear LS problems can be solved analytically. The WLS problem is then solved iteratively by alternately solving each of these four linear LS problems with appropriately updated weighting functions.

II. PROBLEM

The modified structure of an FIR filter synthesized using the FRM technique, introduced in [11], is shown in Fig. 2. If $F(z)$

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Fig. 2. The Modified FRM structure.

Fig. 3. Frequency responses of (a) $F_a(z^{M_a})$ and $F_1(z^{M_1})$; (b) $F_2(z^{M_2})$; (c) $F_{ma}(z)$ and $F_{mc}(z)$; (d) $F(z)$.

is the z -transform transfer function of the system, then, from Fig. 2

$$F(z) = G_1(z)F_{ma}(z) + G_2(z)F_{mc}(z)$$

where

$$G_1(z) = G_3(z)F_2(z^{M_2})$$

$$G_3(z) = z^{-\frac{(l_a-1)M_a}{2} - \frac{(l_1-1)M_1}{2}} - F_a(z^{M_a})F_1(z^{M_1})$$

$$G_2(z) = z^{-\frac{(l_a-1)M_a}{2} - \frac{(l_1-1)M_1}{2} - \frac{(l_2-1)M_2}{2}} - G_1(z)$$

$$F_a(z^{M_a}) = \sum_{n=0}^{l_a-1} h_a(n)z^{-nM_a}$$

$$F_1(z^{M_1}) = \sum_{n=0}^{l_1-1} h_1(n)z^{-nM_1}$$

$$F_2(z^{M_2}) = \sum_{n=0}^{l_2-1} h_2(n)z^{-nM_2}$$

$$F_{ma}(z) = \sum_{n=0}^{l_{ma}-1} h_{ma}(n)z^{-n}$$

$$F_{mc}(z) = \sum_{n=0}^{l_{mc}-1} h_{mc}(n)z^{-n}.$$

The frequency response magnitudes $|F_a(e^{j\omega M_a})|$, $|F_1(e^{j\omega M_1})|$, $|F_2(e^{j\omega M_2})|$, $|F_{ma}(e^{j\omega})|$ and $|F_{mc}(e^{j\omega})|$, of these filters are shown in Fig. 3. If $G_1(z)F_{ma}(z)$ and $G_2(z)F_{mc}(z)$ have the same phase delay, the resulting frequency response magnitude, $|F(e^{j\omega})|$, of $F(z)$ is shown

in Fig. 3(d). The transition width of $F(z)$ is narrower than that of $F_a(z)$ by a factor of M_a . With this new structure, the restriction on M_a to be a multiple of M_1 , which is required for an IFIR-FRM approach [12], is relaxed. Furthermore, an additional decoupling filter $F_2(z^{M_2})$ is inserted to balance the complexities of the two masking filters. The requirement on the masking filters $F_{ma}(z)$ and $F_{mc}(z)$ is, therefore, further relaxed (For details, see [11]).

Given the passband edge w_p , the stopband edge w_s , the permitted maximum passband ripple δ_p , and the stopband ripple δ_s of a desired filter, the interpolation ratios M_a, M_1 , and M_2 for the subfilters F_a, F_1 , and F_2 , respectively, can be optimized (for details, see [11]). The band edges for the subfilters F_a, F_1, F_2, F_{ma} and F_{mc} can be derived using the equations in [1], [11], [20]. Therefore, the general problem of designing an FIR filter synthesized using the modified FRM structure introduced in [11] may be formulated as: Given the band edges w_p and w_s , the permitted maximum band ripples δ_p and δ_s , and the interpolation ratios M_a, M_1 and M_2 , find, respectively, the filter lengths l_a, l_1, l_2, l_{ma} and l_{mc} of the subfilters of the system, and coefficients $\mathbf{h}_a, \mathbf{h}_1, \mathbf{h}_2, \mathbf{h}_{ma}$ and $\mathbf{h}_{mc}$ of the subfilters F_a, F_1, F_2, F_{ma} and F_{mc} , such that the maximum error, $E(w)$, is minimized over the whole region, where $E(w)$ is defined by

$$E(w) = W(w)|D(w) - F(w)|. \quad (1)$$

Here, $W(w)$ and $D(w)$ are the frequency weighting function and the desired response of the system, respectively. In the passband, $W(w)$ and $D(w)$ are equal to one. In the stopband, $W(w)$ is equal to δ_p/δ_s and $D(w)$ is equal to zero.

This minimax problem is to be solved as a WLS problem. More specifically, we minimize the objective function

$$\epsilon = \sum_{w \in \Xi} B^2(w)|E(w)|^2 \quad (2)$$

where Ξ is a dense set of frequencies uniformly distributed in $[0, w_p] \cup [w_s, 2\pi]$, and $B(w)$ is a weighting function. Let this WLS problem be referred to as Problem (P).

III. SOLUTION METHOD

Suppose that the filter lengths $l_a, l_1, l_2, l_{ma}, l_{mc}$, of the subfilters are determined. Then, Problem (P) reduces to finding the coefficients $\mathbf{h}_a, \mathbf{h}_1, \mathbf{h}_2, \mathbf{h}_{ma}$ and $\mathbf{h}_{mc}$ of the subfilters F_a, F_1, F_2, F_{ma} and F_{mc} , respectively, such that the objective function (2) is minimized. The objective function is highly nonlinear with respect to the coefficients of the subfilters. Thus, it is very difficult to minimize the objective function using any direct optimization approach. In [21], an unconstrained WLS algorithm is used to jointly optimize the subfilters of an FIR filter synthesized using the original one-stage FRM structure. In each step, a quasi-Newton method is employed, where the Davidon–Fletcher–Powell (DFP) formula is used to update the search direction. The objective function is highly nonlinear and nonconvex with respect to the coefficients. Furthermore, its Hessian matrix is dense. Thus, the method can be computationally expensive for large problems. The optimization problem addressed in [21] is for the one-stage FRM structure.

In contrast, the optimization problem described in this paper is for the modified FRM structure, where the single band-edge shaping filter in the one-stage FRM structure is replaced by a cascade combination of three subfilters. Clearly, the optimization problem addressed in this paper is much more complicated than that considered in [21]. Therefore, the direct optimization method proposed in [21] is unlikely to work for this problem.

By careful examination of Problem (P), we see that it can be conveniently decomposed into several subproblems, as certain subsets of $\{\mathbf{h}_a, \mathbf{h}_1, \mathbf{h}_2, \mathbf{h}_{ma}, \mathbf{h}_{mc}\}$ affect fairly distinct parts of the frequency range [1]. In the part of the frequency range where $\mathbf{h}_{ma}$ and $\mathbf{h}_{mc}$ are both equal to 1 or 0, the error is determined mainly by $\mathbf{h}_{ma}$ and $\mathbf{h}_{mc}$, while the effect of $\{\mathbf{h}_a, \mathbf{h}_1, \mathbf{h}_2\}$ is of secondary importance. On the other hand, in the frequency range where $|\mathbf{h}_{ma}(w) - \mathbf{h}_{mc}(w)|$ is large, (i.e., when $w \approx w_p$ or $w \approx w_s$), the error is very much determined by $\{\mathbf{h}_a, \mathbf{h}_1, \mathbf{h}_2\}$. We thus divide $\{\mathbf{h}_a, \mathbf{h}_1, \mathbf{h}_2, \mathbf{h}_{ma}, \mathbf{h}_{mc}\}$ into two subsets, $\{\mathbf{h}_a, \mathbf{h}_1, \mathbf{h}_2\}$ and $\{\mathbf{h}_{ma}, \mathbf{h}_{mc}\}$. The influence of each subset is largely restricted to the corresponding parts of the frequency range. Their effects are mutually compensated only on relatively small intervals of the frequency range. We therefore propose an alternating variable approach based on these two groups of coefficients to solve Problem (P). For this, let us first define the following two optimization subproblems.

Problem (P₁): The coefficients $\mathbf{h}_a, \mathbf{h}_1$ and $\mathbf{h}_2$ of the objective function (2) are taken as fixed. Let the corresponding new objective function be denoted as $\epsilon_1((\mathbf{h}_{ma}, \mathbf{h}_{mc})|B, \mathbf{h}_a, \mathbf{h}_1, \mathbf{h}_2)$, where B is the weighting function which is to be appropriately updated.

Problem (P₂): The coefficients $\mathbf{h}_{ma}$ and $\mathbf{h}_{mc}$ of the objective function (2) are taken as fixed. Let the corresponding new objective function be denoted as $\epsilon_2((\mathbf{h}_a, \mathbf{h}_1, \mathbf{h}_2)|B, \mathbf{h}_{ma}, \mathbf{h}_{mc})$, where B is the weighting function which is to be appropriately updated.

Problem (P): is solved through solving Problem (P₁) and Problem (P₂) alternately. Problems (P₁) and (P₂) are solved as follows.

Note that Problem (P₁) contains two unknown sets of coefficients $\mathbf{h}_{ma}$ and $\mathbf{h}_{mc}$. These belong to the subfilters F_{ma} and F_{mc} , respectively, which are connected in parallel. As a result, the frequency response magnitude of the overall filter $F(w)$ is linear with respect to the coefficients in $\mathbf{h}_{ma}$ and $\mathbf{h}_{mc}$. Therefore, Problem (P₁) can be solved analytically (details are given in Section III-A). Problem (P₂) contains three unknown sets of coefficients, $\mathbf{h}_a, \mathbf{h}_1$ and $\mathbf{h}_2$ belonging to the subfilters, F_a, F_1 and F_2 , respectively. These subfilters are connected in a cascade fashion. Thus, the frequency response magnitude of the overall filter $F(w)$ is nonlinear with respect to the coefficients, $\mathbf{h}_a, \mathbf{h}_1$ and $\mathbf{h}_2$. To overcome the complexity in the minimization on the objective function (2) with respect to the coefficients, $\mathbf{h}_a, \mathbf{h}_1$ and $\mathbf{h}_2$, we decompose Problem (P₂) further into three subproblems and then use an alternating variable method to solve it. Let us define the following three optimization subproblems.

Problem (P₂₁): The coefficients $\mathbf{h}_1, \mathbf{h}_2, \mathbf{h}_{ma}$ and $\mathbf{h}_{mc}$ of the objective function (2) are taken as fixed. Let the corresponding new objective function be denoted as $\epsilon_{21}(\mathbf{h}_a|B, \mathbf{h}_1, \mathbf{h}_2, \mathbf{h}_{ma}, \mathbf{h}_{mc})$, where B is the weighting function which is to be appropriately updated.

Problem (P₂₂): The coefficients $\mathbf{h}_a, \mathbf{h}_2, \mathbf{h}_{ma}$ and $\mathbf{h}_{mc}$ of the objective function (2) are taken as fixed. Let the corresponding new objective function be denoted as $\epsilon_{22}(\mathbf{h}_1|B, \mathbf{h}_a, \mathbf{h}_2, \mathbf{h}_{ma}, \mathbf{h}_{mc})$, where B is the weighting function which is to be appropriately updated.

Problem (P₂₃): The coefficients $\mathbf{h}_a, \mathbf{h}_1, \mathbf{h}_{ma}$ and $\mathbf{h}_{mc}$ of the objective function (2) are taken as fixed. Let the corresponding new objective function be denoted as $\epsilon_{23}(\mathbf{h}_2|B, \mathbf{h}_1, \mathbf{h}_2, \mathbf{h}_{ma}, \mathbf{h}_{mc})$, where B is the weighting function which is to be appropriately updated.

Problem (P₂) is then solved through solving a sequence of Problems (P₂₁), (P₂₂) and (P₂₃) alternately.

There are now two instances of an alternating variable search. The first one is to solve Problem (P) via Problems (P₁) and (P₂). The second one is to solve Problem (P₂) via Problems (P₂₁), (P₂₂) and (P₂₃). However, this approach is computationally time-consuming. Thus, we propose another technique, where the alternating search within Problem (P₂) is incorporated into the alternating search between Problems (P₁) and (P₂). It may be described as follows:

Data: Choose a positive integer m , and a small positive real number Δ .

- Step 1) Find an initial filter $\{*, \mathbf{h}_1^1, \mathbf{h}_2^1, \mathbf{h}_{ma}^1, \mathbf{h}_{mc}^1\}$ (without $\mathbf{h}_a^1$) by designing the subfilters, $F_1(w), F_2(w), F_{ma}(w)$ and $F_{mc}(w)$, separately. Set the initial weighting function, $B_1(w) = 1$, and iteration number, $k = 1$. Set $i = 0$ and $j = 0$.
- Step 2) Minimize $\epsilon_{21}(\mathbf{h}_a|B_k, \mathbf{h}_1^k, \mathbf{h}_2^k, \mathbf{h}_{ma}^k, \mathbf{h}_{mc}^k)$ with respect to the coefficients $\mathbf{h}_a$. Denote the optimal coefficients obtained as $\mathbf{h}_a^k$.
- Step 3) Minimize $\epsilon_1((\mathbf{h}_{ma}, \mathbf{h}_{mc})|B_k, \mathbf{h}_a^k, \mathbf{h}_1^k, \mathbf{h}_2^k)$ with respect to the coefficients $\mathbf{h}_{ma}$ and $\mathbf{h}_{mc}$. Denote the optimal coefficients obtained as $\mathbf{h}_{ma}^k$ and $\mathbf{h}_{mc}^k$. If $j = 0$, goto Step 4. If $j = 1$, goto Step 5. If $j = 2$, set $i = i + 1$. If $i = m$, goto Step 6. Otherwise, set $j = 0$ and goto Step 2).
- Step 4) Minimize $\epsilon_{22}(\mathbf{h}_1|B_k, \mathbf{h}_a^k, \mathbf{h}_2^k, \mathbf{h}_{ma}^k, \mathbf{h}_{mc}^k)$ with respect to the coefficients $\mathbf{h}_1$. Denote the optimal coefficients obtained as $\mathbf{h}_1^k$. Set $j = 1$. Goto Step 3).
- Step 5) Minimize $\epsilon_{23}(\mathbf{h}_2|B_k, \mathbf{h}_a^k, \mathbf{h}_1^k, \mathbf{h}_{ma}^k, \mathbf{h}_{mc}^k)$ with respect to the coefficients $\mathbf{h}_2$. Denote the optimal coefficients obtained as $\mathbf{h}_2^k$. Set $j = 2$ and goto Step 3).
- Step 6) Calculate $\delta^k = \max_{w \in [0, w_p] \cup [w_s, \pi]} W(w) |D(w) - F(w)|$. If $|\delta^k - \delta^{k-1}| < \Delta$, where Δ is a prescribed tolerance, then stop. Otherwise, update $B_{k+1}(w) = B_k(w) \bar{\beta}_k(w)$, where $\bar{\beta}_k(w)$ is defined in the next section. Set $k = k + 1, i = 0, j = 0$ and go to Step 2).

From our extensive numerical simulation study, we observe that for the filter design using the above procedure, the filter lengths $l_a, l_1, l_2, l_{ma}, l_{mc}$ of the subfilters F_a, F_1, F_2, F_{ma} and F_{mc} are estimated to be approximately 100, 100, 85, 65 and 65 percent, respectively, of those designs obtained by the technique in [11].

Remark: In Step 6) of the algorithm, the weighting function is updated every m iterations. From our extensive numerical simulation experience, we observe that it is sufficient to choose m to be between 3 and 5. The tolerance Δ determines the terminating condition for the algorithm.

A. Solution of Problem (P_1)

Note that the optimality condition for the coefficients $\mathbf{h}_{ma}$ and $\mathbf{h}_{mc}$ of Problem (P_1) is easily obtained by setting the gradient of the objective function ϵ_1 with respect to these coefficients to zero. This gives rise to the following system of linear equations:

$$RUX = RH \quad (3)$$

where

$$U = \begin{bmatrix} U_1(w_1) & U_2(w_1) \\ U_1(w_2) & U_2(w_2) \\ \vdots & \vdots \end{bmatrix}$$

$$U_1(w) = \{G_1(w)\text{trig}_{ma}(w, 1), G_1(w)\text{trig}_{ma}(w, 2), \dots\}$$

$$U_2(w) = \{G_2(w)\text{trig}_{mc}(w, 1), G_2(w)\text{trig}_{mc}(w, 2), \dots\} \quad (4)$$

where $\text{trig}_{ma}(w, i)$ and $\text{trig}_{mc}(w, i)$ are appropriate trigonometric functions for subfilters $F_{ma}(w)$ and $F_{mc}(w)$, respectively, and

$$X = \{h_{ma}(1), h_{ma}(2), \dots, h_{mc}(1), h_{mc}(2), \dots\}^T \quad (5)$$

$$H = \{D(w_1), D(w_2), \dots\}^T. \quad (6)$$

R is the weighting matrix given by

$$R = \begin{bmatrix} B(w_1) & \circ & \\ & B(w_2) & \\ \circ & & \ddots \end{bmatrix} \quad (7)$$

and $\{w_i\}$ is a dense set of frequencies uniformly distributed in $[0, w_p] \cup [w_s, \pi]$. The least-squares solution of (3) is given by

$$X = (U^T R^2 U)^{-1} U^T R^2 H. \quad (8)$$

B. Solutions of Problems (P_{21}), (P_{22}) and (P_{23})

Note that the optimality conditions for Problems (P_{21}), (P_{22}), and (P_{23}) are easily obtained by setting the gradients of the objective functions ϵ_{21} , ϵ_{22} and ϵ_{23} with respect to their associated coefficients equal to zero. They give rise to the following form of linear system:

$$RVY = RG \quad (9)$$

where

$$V = \begin{bmatrix} V_1(w_1) & V_2(w_1) & \dots \\ V_1(w_2) & V_2(w_2) & \dots \\ \vdots & \vdots & \ddots \end{bmatrix} \quad (10)$$

$$Y = \{Y(1), Y(2), \dots\}^T \quad (11)$$

$$= \{g(w_1), g(w_2), \dots\}^T \quad (12)$$

and the weighting matrix R is defined by (7).

The least-squares solution of (9) is given by

$$Y = (V^T R^2 V)^{-1} V^T R^2 G. \quad (13)$$

For Problem (P_{21}), $V_i(w) = (F_{ma}(w) - F_{mc}(w))F_1(w)F_2(w)\text{trig}_a(M_a w, i)$, where $\text{trig}_a(w, i)$ are appropriate trigonometric functions for subfilter $F_a(w)$, $Y(i) = h_a(i)$, and $g(w) = F_{mc}(w) + F_2(w)(F_{ma}(w) - F_{mc}(w)) - D(w)$. For Problem (P_{22}), $V_i(w) = (F_{ma}(w) - F_{mc}(w))F_a(w)F_2(w)\text{trig}_1(M_1 w, i)$, where $\text{trig}_1(w, i)$ are appropriate trigonometric functions for subfilter $F_1(w)$, $Y(i) = h_1(i)$, and $g(w) = F_{mc}(w) + F_2(w)(F_{ma}(w) - F_{mc}(w)) - D(w)$.

For Problem (P_{23}), $V_i(w) = (F_{ma}(w) - F_{mc}(w))(1.0 - F_a(w)F_1(w))\text{trig}_2(M_2 w, i)$, where $\text{trig}_2(w, i)$ are appropriate trigonometric functions for subfilter $F_2(w)$, $Y(i) = h_2(i)$, and $g(w) = F_{mc}(w) - D(w)$.

The updated error weighting function at each iteration k is given [22] by $B_{k+1}(w_n) = B_k(w_n)\beta_k(w_n)$

$$\text{with } \beta_k(w_n) = \frac{|E_k(w_n)|}{\sum_m B_k(w_m)|E_k(w_m)|}.$$

In order to prevent $\beta_k(w_n)$ from getting too close to zero, we introduce a lower bound, γ , similar to the one used in [22]

$$\bar{\beta}_k(w) = \begin{cases} \beta_k(w) & \text{if } \beta_k(w) \geq \gamma \\ \gamma & \text{otherwise,} \end{cases}$$

$$\text{where } \gamma \leq v = \frac{\min\{\mathbf{h}_a, \dots, \mathbf{h}_{mc}\} \max_{w \in [0, w_p] \cup [w_s, \pi]} E(w)}{\sum_m B_k(w_m)|E_k(w_m)|}.$$

To accelerate the convergence, $\bar{\beta}_k(w_n)$ is made proportional to a higher power of $\bar{\beta}_k(w_n)$ as given by [22]

$$\bar{\bar{\beta}}_k(w_n) = \left(A_k \frac{\bar{\beta}_k(w_n)}{W(w_n)} \right)^\theta.$$

Here, $W(w)$ is the minimax weighting function as indicated before, and A_k is a constant which is selected to maintain a reasonable dynamic range for the value of $B_k(w_n)$. As in [22], the constant A_k may be selected as the average value of $D(w_n)/B_k(w_n)$ over all n . The constant θ affects the rate of convergence, see [22] for details. We take the value of θ to be between 1 and 1.7 as in [22].

IV. DESIGN EXAMPLE

As in [11] and [12], a linear phase low-pass FIR filter with the passband frequencies $0 \leq w/2\pi \leq 0.3$ and the stopband frequencies $0.301 \leq w/2\pi \leq 0.5$ is considered. The allowable maximum passband and stopband ripples are $\delta_p = 0.01$ and $\delta_s = 0.0001$, respectively. We use the same interpolation ratios as in [11]: $M_a = 39$, $M_1 = 13$, and $M_2 = 4$. For WLS Chebyshev solution, we use 10 001 sample points in the frequency range. Our method takes 364 iterations to meet the minimax criteria with Problems (P_1), (P_{21}), (P_1), (P_{22}), (P_1), and (P_{23}) being solved alternately 4 times at each iteration. All computations are performed on an SGI Origin 2000. The computer used 13.1 minutes to complete the calculation. The filter lengths of F_a , F_1 , F_2 , F_{ma} , and F_{mc} used are 83, 21, 49, 38, and 16, respectively. 105 multipliers are needed. The maximum

Fig. 4. Frequency response of the overall filter.

Fig. 5. Frequency response of $F_{ma}(w)$ and $F_{mc}(w)$. Fig. 6. Frequency responses of $F_a(M_a w)$ (solid line), $F_1(M_1 w)$ (dashed line) and $F_2(M_2 w)$ (dotted line).

passband and stopband ripples achieved are $\delta_p = 0.00835$ and $\delta_s = 0.0000835$, respectively. For comparison, the filter lengths of F_a , F_1 , F_2 , F_{ma} , and F_{mc} used in [11] to meet the requirements are 83, 21, 57, 56, 24, respectively. 122 multipliers are needed. The filter designed by using our approach yields a

14% savings in the number of multipliers required when compared with the filter produced in [11]. The frequency response of the filter and subfilters designed by the proposed method are depicted in Figs. 4–6, respectively.

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